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A dual-threshold approach for the dynamics of bi-polarization in signed networks with communities

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ABSTRACT

The intensified global bi-polarization has been threatening social stability and democracy in recent years, with social networks of various forms playing a key role in shaping opinion dynamics. Understanding how individual and collective opinions form, change, and spread is crucial for mitigating bi-polarization. However, existing research rarely investigates the effects of signs on edges and their distributional characteristics among communities. This study introduces a dual-threshold opinion dynamics model in signed networks with communities to identify and analyze the impact of edge heterogeneity and its distribution on assimilation and repulsion social influences in network opinion evolution. It reveals that repulsive social influences introduced by negative edges cause assimilative social influences from positive edges to increase bi-polarization in a non-monotonic manner. The distribution of initial opinions and the tendency of intra-community node connections significantly affect the degree of bi-polarization. Different initial opinions and moderate inter-community connections can mitigate bi-polarization. Additionally, the density of positive inter-community connections has a non-monotonic effect on bi-polarization, increasing the likelihood of network consensus. This study uncovers the complex dynamics of opinion evolution, enhances understanding of how social influence and network structures shape opinion dynamics, and offers a broader perspective for effectively managing and influencing the evolution of public opinion.

1. Introduction

Polarization, typically refers to social and political divisions, increases antagonism between different groups and has been threatening social stability and democracy globally in recent years. The Global Risk Report 2024 highlights social polarization as one of the top three major risks due to its profound impact on societal cohesion and governance [1]. Deepening divisions erode institutional trust, fueling political extremism and the risk of social unrest. These dynamics pose serious threat to social stability and the democratic process, and undermines the foundation of collaborative governance and peaceful coexistence. The debate over how social networks influence opinion polarization has always existed. On the one hand, social networking platforms provide new channels for users to express opinions and interact, but also amplify the effects of information bubbles and echo chambers, leading to reduced contact between different groups and intensified opinion conflicts [2–4]. On the other hand, the recommendation algorithms of social networking platforms can reduce exposure to opposing opinions, potentially preventing further polarization [5,6]. Studies on opinion dynamics have attracted widespread attention, bringing together multidisciplinary efforts to explore how individual and collective opinions form, change, and spread over time [7–11]. Significant progress has been made in understanding opinion polarization within this context.

In this study, bi-polarization is defined as a sharp division of opinions within a group on a specific issue into two opposing camps, resulting in the erosion of moderate perspectives and heightened social conflict [12,13]. This differs from the group polarization phenomenon described in social psychology, where a group makes decisions that are more extreme than the initial inclinations of its individual members [14]. Existing opinion dynamics models that explore bi-polarization, such as iterative weighted averaging and stubborn agent models, primarily focus on opinion convergence but struggle to explain the phenomenon of increasing bi-polarization of individual opinions [8,15–17]. Subsequently, researchers introduced models that combine negative social influence and persuasion theory, revealing extreme trends in interactions between individuals with similar attitudes [18–21]. Additionally, models incorporating internal opinions

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and discrete expressions lead to convergence or bi-polarization through neighborhood imitation mechanisms [22]. The latest reinforcement learning models, combined with game theory, propose explanations for the generation and maintenance of bi-polarization through the interaction of tight groups and the gating effect of structural holes, thereby enriching the understanding of the mechanisms behind opinion bi-polarization [23].

Despite the noticeable progresses made in explaining the phenomenon of opinion bi-polarization, existing opinion dynamics research often overlooks the complexity of negative relationships and antagonistic interactions in social networks. Such antagonistic relationships exist in numerous real-world systems, particularly in the social, biological and information fields. For instance, antagonistic relationships can emerge in diverse contexts, such as between political opponents in social networks, predators and prey in ecosystems, or rival companies in information networks. These adversarial interactions can profoundly influence the dynamics and evolution of these systems across various domains [24–26]. Signed networks are used to describe these antagonistic relationships [27,28]. However, when exploring the issue of bi-polarization in signed networks, most research has primarily focused on the balance characteristics of network structures and the conditions under which bi-polarization occurs [29–31]. Previous research has primarily focused on the influence of network structure, particularly structural balance, on opinion dynamics in signed networks. For instance, Altafini et al. demonstrated that structurally balanced networks tend to converge towards consensus [32], while Shi et al. relaxed this condition to quasi-structural balance [33]. Cisneros-Velarde et al. incorporated the boomerang effect, revealing more complex opinion behaviors in unbalanced networks [34]. Lee et al. examined consensus and polarization in fully connected, structurally balanced networks [35]. Talaga et al. developed methods to assess the degree of balance at different scales [30]. However, a comprehensive understanding of how edge heterogeneity within these networks impacts opinion evolution remains unexplored.

At the same time, social networks often exhibit community structures, which may significant impact the evolution of opinions and the formation of bi-polarization. Peng et al. constructed a bounded confidence opinion dynamics model and introduced a label propagation algorithm to study the relationship between public opinion evolution and community division from a micro perspective [36]. Si et al. used the Sznajd update rule to simulate individuals' behavior when encountering strangers in different communities, studying the impact of implicit community structures in networks on opinion evolution [37]. Ru et al. introduced the Sznajd model and explored the factors influencing consensus formation in network opinions in scale-free networks with adjustable strength of community structures [38]. Fang studied the impact of communication structure on opinion evolution in social networks composed of strongly connected communities and non-community outsiders [39]. In addition, some researchers have considered applying opinion dynamics to solve the problem of community detection and have proposed a series of community detection methods based on opinion dynamics models [40]. These studies indicate a close relationship between community structure and opinion dynamics. However, existing research have not effectively integrated community structures and features of signed networks to explore how heterogeneous edges and their distribution in the network affect opinion evolution and the mechanisms underlying the formation of bi-polarization. Based on the signed network, the community structure is characterized by predominantly positive relationships within groups and predominantly negative relationships between groups [41]. This structural feature is particularly important in studying the impact of edge heterogeneity and its distribution on opinion evolution in the network.

Building upon the signed community structure models by Yang et al. [41] and Li et al. [42], this study investigates the dynamic evolution of continuous opinions in signed networks with communities.

Table 1
Application parameter description.

Parameter	Description
N	The number of individuals included in the network
k	The average degree of nodes in the network
C	The number of sub-communities included in the signed network
p_i	The probability of each individual connecting to others within the same community
$1-p_i$	The probability of an individual connecting to individuals in different communities
p^+_{cc}	The ratio of positive edges between communities
p^-_c	The ratio of negative edges within communities
E	The matrix of relationships between individuals in the network
$O(t)$	The set of opinions in the network at time t
$e_{ij}=+1$	Node $i\in N$ and node $j\in N$ have a positive edge between them
$e_{ij}=-1$	Node $i\in N$ and node $j\in N$ have a negative edge between them
ϵ_T	The assimilative threshold identifies similar opinions, triggering assimilative social influence
ϵ_R	The repulsive threshold identifies dissenting opinions, triggering repulsive social influence
$O^+_i(t)$	The set of neighbors that exert assimilative social influence on individual i
$O^-_i(t)$	The set of neighbors that exert repulsive social influence on individual i

Employing a dual-threshold opinion interaction mechanism [43], we examine how assimilative and repulsive social influences, modulated by edge attributes, shape opinion formation. Our analysis focuses on understanding the impact of edge signs and their distribution on opinion dynamics within different community-signed network topologies.

By elucidating these mechanisms, we can uncover how edge heterogeneity and distribution within community-structured signed networks either intensify or alleviate opinion polarization in social networks. The remainder of this paper is organized as follows: Section 2 delves into the proposed individual opinion interaction mechanisms within the community-signed network. Section 3 presents numerical simulation results and their implications. Finally, Section 4 summarizes key findings and conclusions.

2. Methods

To better explore the impact of community structure in signed networks on opinion evolution, and to analyze the combined effect of heterogeneous edges on individual opinion updates, this study constructs a dual-threshold opinion dynamics model based on the signed network with community structures. The following will detail the steps involved in constructing this model.

2.1. The construction of signed network with communities

The community structure in signed networks is defined by positive edges connecting nodes within the same group and negative edges primarily linking nodes between different groups. The model leverages parameters such as network size N, community count C, average node degree k, and connection probabilities p_i, p^+_{cc}, and p^-_c to delineate community affiliations and inter-community sentiments. The specific parameters are described in Table 1.

The steps for constructing a signed network with community structure are as follows:

Step 1: *Community labeling*. Nodes in the network are divided into C predefined communities, where N is the total number of nodes and $\left\lfloor \frac{N}{C} \right\rfloor$ represents the quotient of nodes assigned to each community. The nodes are first sorted by their indices: the first $\left\lfloor \frac{N}{C} \right\rfloor$ nodes are assigned to community 1, the next $\left\lfloor \frac{N}{C} \right\rfloor$ nodes to community 2, and so

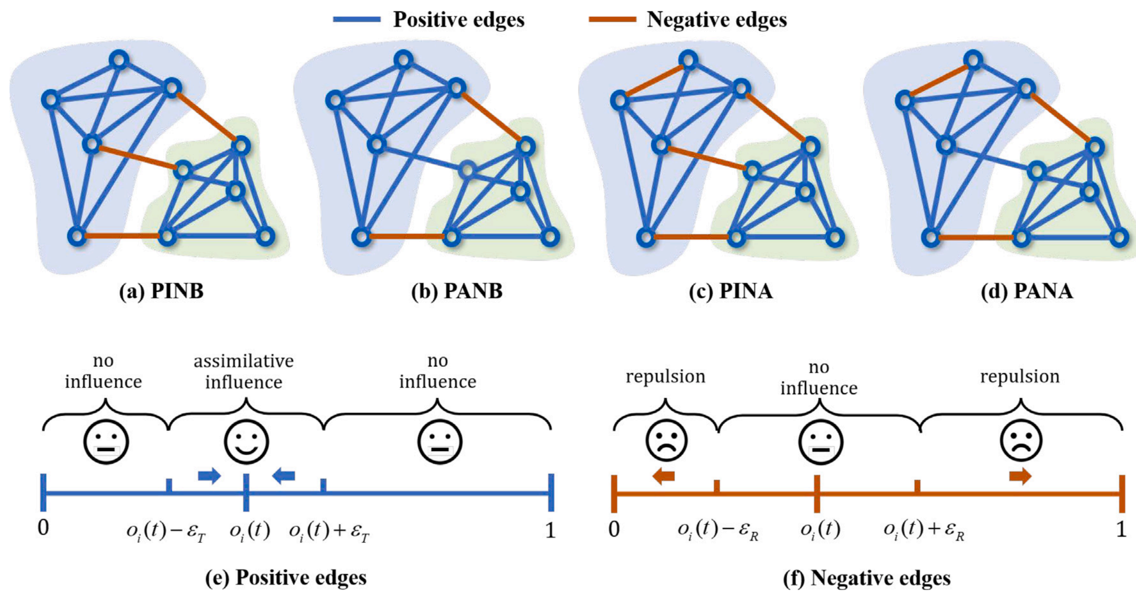

Fig. 1. Different types of signed networks with communities and opinion interaction mechanism.

on, until all nodes are assigned. If N is not perfectly divisible by C, the remaining nodes are distributed one by one among the first $N \mod C$ communities, resulting in these communities having $\left\lfloor \frac{N}{C} \right\rfloor + 1$ nodes, while the remaining communities contain $\left\lfloor \frac{N}{C} \right\rfloor$ nodes. This approach ensures a proportional and balanced distribution of nodes across the C communities.

Step 2: *Intra-community connections*. Based on the average node degree k and connection probability p_i, nodes establish connections within the same community proportional to kp_i. Additionally, within sub-communities, $kp_ip_c^-$ determines the number of negative connections ($e_{ij} = -1$). These negative connections are randomly assigned within each community, while the remaining connections are assigned positive attributes ($e_{ij} = +1$), calculated as $kp_i(1 - p_c^-)$.

Step 3: *Inter-community links*. Based on the number of connections that nodes make to nodes in different communities, $k(1 - p_i)$, nodes from other communities are randomly selected based on the calculated number of inter-community connections. The number of positive inter-community connections is determined by the parameter p_{cc}^+, calculated as $k(1 - p_i)p_{cc}^+$. Randomly select the corresponding number of connections and assign them positive attributes ($e_{ij} = +1$). The remaining inter-community connections are assigned negative attributes ($e_{ij} = -1$), calculated as $k(1 - p_i)(1 - p_{cc}^+)$.

Step 4: *Parameter adjustment*. Adjust p_{cc}^+ and p_c^- to explore different configurations of signed community structures.

According to the above steps, once parameters N, k, C are determined, the type of community structure in the signed network depends on the parameters p_i, p_{cc}^+, and p_c^-. By adjusting these parameters, the signed network can be categorized into four types based on the proportion of positive and negative edges within and between communities. Fig. 1 illustrates the four typical types of signed networks with community structures using a network of 10 individuals containing two subcommunities as an example.

(a) ***PINB***. In this network, positive edges exclusively exist within communities, while negative edges exist strictly between communities, as depicted in Fig. 1(a). In this case, $p_{cc}^+ = 0$, $p_c^- = 0$.

(b) ***PANB***. In this network, positive edges are exclusively within communities, while both positive and negative edges exist between communities, as depicted in Fig. 1(b). In this case, $p_{cc}^+ = 0.1$, $p_c^- = 0$.

(c) ***PINA***. In this network, negative edges exclusively connect between communities, while both positive and negative edges are present within communities, as depicted in Fig. 1(c). In this case, $p_{cc}^+ = 0$, $p_c^- = 0.1$.

(d) ***PANA***. In this network, a specific proportion of positive and negative edges will be observed both within communities and between communities, as depicted in Fig. 1(d). In this case, $p_{cc}^+ = 0.1$, $p_c^- = 0.1$.

2.2. Opinion update rule

Given a signed network $G(N, E, O(t))$, where $N = \{1, 2, \ldots, n\}$, the edges E can either be positive ($e_{ij} = 1$) or negative ($e_{ij} = -1$). For simplicity, we treated each edge as undirected in this paper, i.e., $e_{ij} = e_{ji}$. The opinion of the ith individual at time t is denoted by $o_i(t)$, where $o_i(t) \in [0, 1]$.

The steps for the opinion interaction mechanism in a signed network with communities are as follows:

Step 1: *Filtering social influence types*. Two thresholds, ϵ_T and ϵ_R, are established to filter network neighbors. Individuals are influenced by positive edges (friends with similar opinions), leading to assimilative social influence, and by negative edges (enemies with differing opinions), resulting in repulsive social influence, as depicted in Fig. 1(e) and (f). The sets of neighbors exerting assimilative and repulsive social influences on individual i at a given moment t are described as follows:

$$\begin{aligned} O_i^+(t) &= \{j \mid |o_i(t) - o_j(t)| \le \epsilon_T \cap e_{ij} = +1\} \\ O_i^-(t) &= \{j \mid |o_i(t) - o_j(t)| \ge \epsilon_R \cap e_{ij} = -1\} \end{aligned} \tag{1}$$

Step 2: *Calculating influence weights*. In the signed networks with communities, individuals' opinions are simultaneously attracted to influential friends and repelled by influential enemies. The strength of this influence is determined by the proportion of influential neighbors (friends or enemies) among all neighbors. This role is captured by the judgmental influence weight function, $\omega_i(t)$, a parameter derived from the network structure itself.

$$\begin{aligned} \omega_i(t) &= \delta(e_{ij} = +1) \cdot \delta(|o_i(t) - o_j(t)| \le \epsilon_T) \\ &\quad + \delta(e_{ij} = -1) \cdot \delta(|o_i(t) - o_j(t)| \ge \epsilon_R) \end{aligned} \tag{2}$$

Here, $\delta(\text{true}) = 1$ and $\delta(\text{false}) = 0$. Eq. (2) implies that the judgmental influence weight function $\omega_i(t)$ is varying with changes in opinions.

Step 3: ***Opinion iteration***. If the number of neighbors of an individual is not empty, i.e., $n_i > 0$, then the opinion update rule for individuals in the network is given by:

$$o_i(t+1) = o_i(t) + \frac{\lambda}{n_i}\left(\sum_{j \in n_i} \omega_i(t) \cdot e_{ij} \cdot (o_j(t) - o_i(t))\right) \tag{3}$$

If the number of neighbors is $n_i = 0$, the individual does not change opinion over time.

Step 4: ***Update patterns***. It is important to note that the repulsive social influence is constrained by the opinion scale, which spans from 0 to 1. The scope of the opinion update satisfies the following conditions.

$$o_i(t+1) = \begin{cases} 0 & \text{if } x < 0 \\ o_i(t) & \text{if } 0 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases} \tag{4}$$

Additionally, the assumption of synchronized updating of network opinions is adopted, following Hegselmann and Krause [44].

2.3. Experimental setting

2.3.1. Synthetic networks

In the simulation experiments, the parameters of the signed networks with communities are set to $c = 2$, $N = 100$, $k = 8$, $p^+_{cc} = p^-_c = 10\%$ and $p_i = 0.5$. Considering the differences in political background, education level, and social status, this has led to varying tendencies in initial opinions across different communities. The impact of two different initial opinion distributions in sub-communities on the evolution of network opinions was considered: one where the initial opinions in both sub-communities follow a uniform random distribution, and another where the initial opinions in both sub-communities have a biased distribution with opposing tendencies.

The initial opinions $o_{i \in N}(0)$ of the two sub-communities are randomly distributed, with each individual's opinion in the network randomly assigned a value between 0 and 1. In the biased distribution, two distinct sub-communities are formed based on node indices: one sub-community has initial opinions $o_{i \in N/2}(0)$ randomly assigned within the range of 0 to 0.5, indicating a stronger negative inclination towards the event; the other sub-community has initial opinions $o_{i \in N/2}(0)$ randomly assigned within the range of 0.5 to 1, indicating a stronger positive inclination towards the event.

For each type of signed community network structure, 20 networks were randomly generated based on the corresponding generation rule parameters. The opinion iteration step in each network is 1000, ensuring that the opinion evolution reaches a steady state. Each analysis plot shows the average results of these 20 evolutionary trials.

2.3.2. Empirical networks

The study findings of the model are further validated using two real-world signed networks. The first is the ***Gahuku-Gama tribal network***, which is based on the study of the highland cultures of New Guinea [45]. In this network, nodes represent sub-tribes, positive edges indicate political alliances between sub-tribes, and negative edges represent hostile relationships between sub-tribes. The specific connections between nodes in the Gahuku-Gama tribal network are illustrated in supplement information Fig. S4. The modularity of this tribal network is calculated to be 0.431. Since most edges between communities are negative and those within communities are positive, the community structure of this network can be classified as the PANB type.

The second network is the ***Slovenian Parliamentary Party Network 1994*** [46]. In this network, nodes represent political parties, and edges indicate the similarity between two parties. A survey conducted by 72 members of the Slovenian National Assembly assessed the relationships between these parties. Positive edges in the network indicate that the parties have similar viewpoints, while negative edges indicate disagreements between the parties. Through statistical calculations, the properties of these two social networks are presented in Table S6. The modularity of the Slovenian parliamentary party network is 0.4547. Since most edges are positive and there are negative edges between communities, the community structure can be classified as the PINA type. The connection data between the 10 parties in the Slovenian Parliament is shown in supplement information Fig. S5.

2.3.3. Evaluation metric

We use the bi-polarization degree $B_p(t)$, as the variance of the opinion distances between all pairs of agents in the population at time t to measure the differences in opinion evolution under different conditions and the extent of opinion fragmentation in the network. This is expressed in formula (5), where $\overline{d(t)}$ represents the average opinion difference between all agents in the group [18].

$$B_p(t) = \frac{4}{N^2} \sum_{i,j}^{i,j \in N} (|o_i(t) - o_j(t)| - \overline{d(t)})^2 \tag{5}$$

When all agents share the same opinion, the bi-polarization degree is zero ($B_p(t) = 0$). Conversely, when the group is divided into two maximally different subgroups, the bi-polarization degree reaches its maximum value $B_p(t) = 1$. A bi-polarization degree between 0 and 1 indicates that the group has not reached consensus and has not achieved maximum bi-polarization.

3. Results

3.1. The impact of types of signed network with communities

3.1.1. Analysis of the role of assimilative social influences

The impact of assimilative social influence induced by positive edges on the outcome of opinion evolution in different community structures was first explored. The types of signed networks with communities structures are constructed based on Section 2.3, and the results of opinion bi-polarization are analyzed when 20 generated networks reach a steady state under different parameter settings. Fig. 2 illustrates the variation in bi-polarization degree across these networks at fixed repulsion thresholds ($\epsilon_R = 1.0, 0.7, 0.5, 0.3$) and varying assimilation thresholds (ϵ_T). From Fig. 2(a), it can be seen that when excluding the effects of repulsive social influence, the signed networks with communities exhibit similar degree of bi-polarization when the network reaches a stable state, even compared to structures without inter-community positive connections. As the assimilation threshold increases, bi-polarization degree decreases gradually until the entire network opinions converges to consensus around $\epsilon_T = 0.5$.

Considering the repulsive social influence caused by negative edges in signature networks with communities, as depicted in Fig. 2(b–d), when the repulsion threshold is set to 0.7 and the assimilation threshold is below 0.2, multiple opinion clusters coexist in the network with weaker bi-polarization. As the assimilation threshold increases from 0.2 to 0.4, the degree of bi-polarization increases, especially noticeable in the PANB structure. When the assimilation threshold exceeds 0.4, the bi-polarization degree significantly decrease in the PANB and PANA communities, while remaining relatively high in the PINA and PINB communities. This suggests that increasing positive connections between communities and enhancing trust in external influences play crucial roles in achieving opinion consensus. With an increase in the assimilation threshold, consensus within sub-communities becomes easier to achieve, while inter-community positive connections provide opportunities for consensus among the majority. However, persistent repulsive social influence causes some individuals to maintain extreme opinions, blocking the formation of complete consensus.

In Figs. 2(c) and (d), when the repulsion thresholds are set to 0.5 and 0.3 respectively, it is observed that in signed networks with

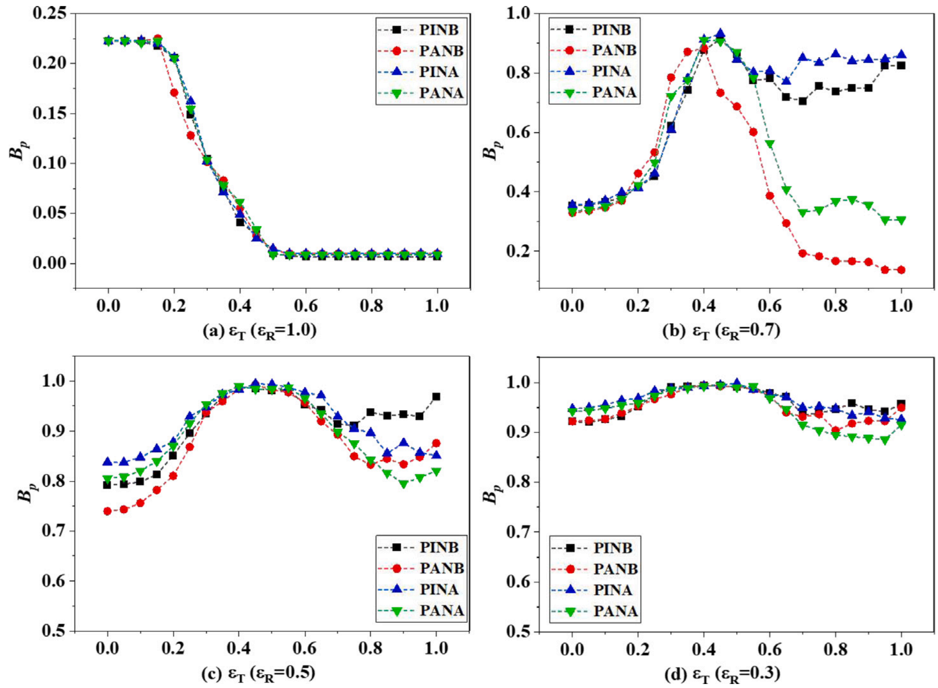

Fig. 2. Variation in the degree of opinion bi-polarization (B_p) in signed networks with community structures, influenced by the assimilation threshold (ϵ_T) at fixed repulsion thresholds (ϵ_R). (a)–(d) illustrate the changes in B_p as ϵ_T ranges from 0 to 1 with intervals of 0.05, while ϵ_R is held constant at 0.3, 0.5, 0.7, and 1.0 respectively.

communities where repulsive social influence is stronger, changes in assimilative social influence intensity have a relatively minor impact on the bi-polarization degree in the stable state of opinion evolution. However, in all four different signed networks with communities, there is a trend where the degree of bi-polarization at the steady state first increases and then decreases as the assimilation threshold increases. From the comparison in Fig. 2, it can be observed that when considering repulsive social influence, the effect of assimilative social influence on opinion evolution in different signed networks with communities is non-monotonic. At the same time, positive connections between communities play a crucial role in the overall consensus of opinions across the network.

3.1.2. Analysis of the role of repulsive social influences

This analysis begins with the construction of various types of signed networks with communities, following the methods outlined in Section 3.1.1. The networks are generated using specific parameters, including fixed assimilation thresholds (ϵ_T) of 0.3, 0.5, 0.7, and 1.0, to examine the impact of repulsion thresholds (ϵ_R) on the degree of bi-polarization at steady state. From the four sub-figures in Fig. 3, it can be seen that as the repulsion threshold increases, the degree of bi-polarization of opinions gradually decreases under different assimilation threshold conditions in the four types of signed networks with communities. Particularly, in Fig. 3(a), it is observed that in PANB and PANA communities, a higher repulsion threshold is required to reduce the degree of bi-polarization, but the reduction rate is relatively faster. In Fig. 3(b)–(d), it is noted that in PINB and PINA communities, even when the repulsion threshold ranges from 0.7 to 0.9, bi-polarization of opinions remains prone to form under higher assimilation threshold conditions. Due to the presence of only repulsive interactions between communities, a high assimilation threshold allows extreme individuals to attract more allies, making bi-polarization likely even under high assimilation thresholds. This indicates that the combined effects of assimilative and repulsive social influences are complex and cannot be simply understood using models focused on single effects. In networks with strong repulsive social influences, maintaining the assimilation threshold at a moderate level (around 0.3) can prevent subgroups from forming extremists, thereby reducing the likelihood of group fragmentation.

3.1.3. Comparative analysis of different community structures in signed networks

To further analyze the differences in opinion evolution within different signed social networks under various threshold combinations, extended experiments were conducted. All combinations of assimilation and repulsion thresholds ranging from 0 to 1, with intervals of 0.05, were explored. Then, 20 independent simulation experiments were carried out, and the average degree of bi-polarization was calculated when opinions evolved to a steady state in different signed networks with community structures. The parameters for constructing these signed networks were consistent with those used in the previous section.

Fig. 4 shows the distribution of the degree of bi-polarization when opinions in the signed networks with random initial opinion distributions evolved to a steady state. The results can be observed that when the repulsion threshold is as high as 0.8, different signed networks with community structures show a common trend: as the assimilation threshold increases, the degree of bi-polarization of network opinions first increases and then decreases. This phenomenon is particularly prominent in the PANB and PANA communities. Notably, when the assimilation threshold is between 0.2 and 0.35, the repulsion threshold also exhibits a non-monotonic effect on the degree of bi-polarization. Additionally, in the PANB community network, even with a high proportion of positive edges (i.e., there are positive edges between communities and no negative edges within communities), the assimilation threshold required to reach the maximum degree of bi-polarization is lower than in the PINB and PINA communities.

In the PINB and PINA community networks, due to the presence of negative edges, the assimilation threshold required for opinion convergence in the network is higher. When the repulsion threshold is close to 1, the degree of bi-polarization in the PINB and PINA communities does not decrease with increasing assimilation threshold. This is because the assimilative interactions within the communities are completely separated, making it difficult to promote consensus across the entire network. In contrast, in the PANB and PANA communities, even with strong repulsive social influence, the assimilative interactions between communities provide the possibility for consensus formation when both the assimilation threshold and repulsion threshold are high. This

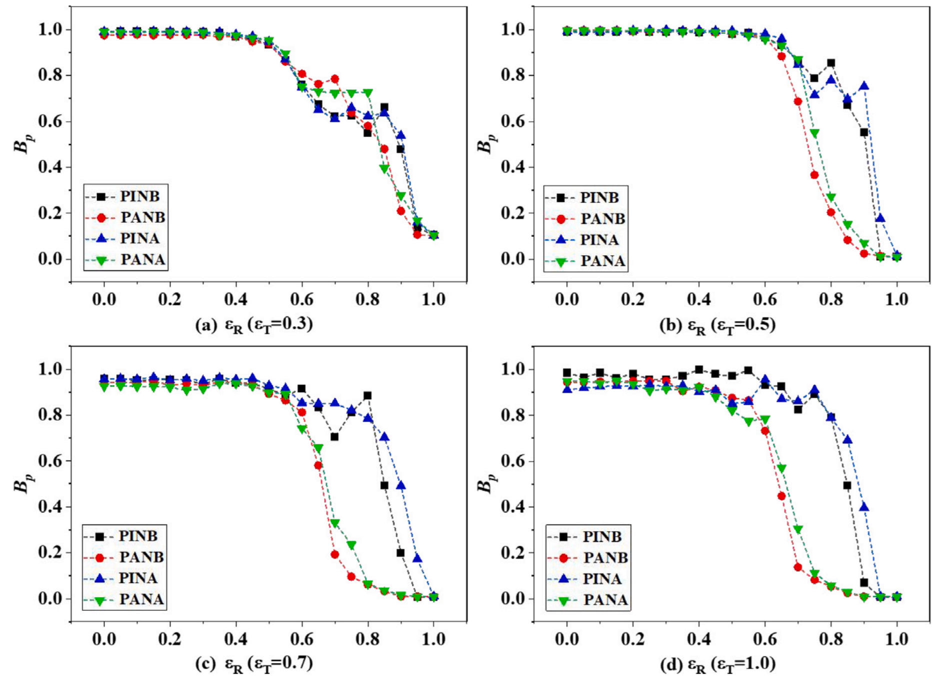

Fig. 3. Variation in the degree of opinion bi-polarization (B_p) in signed networks with community structures, influenced by the repulsion threshold (ϵ_R) at fixed assimilation thresholds (ϵ_T). (a)–(d) illustrate the changes in B_p as ϵ_R ranges from 0 to 1 with intervals of 0.05, while ϵ_T is held constant at 0.3, 0.5, 0.7, and 1.0 respectively.

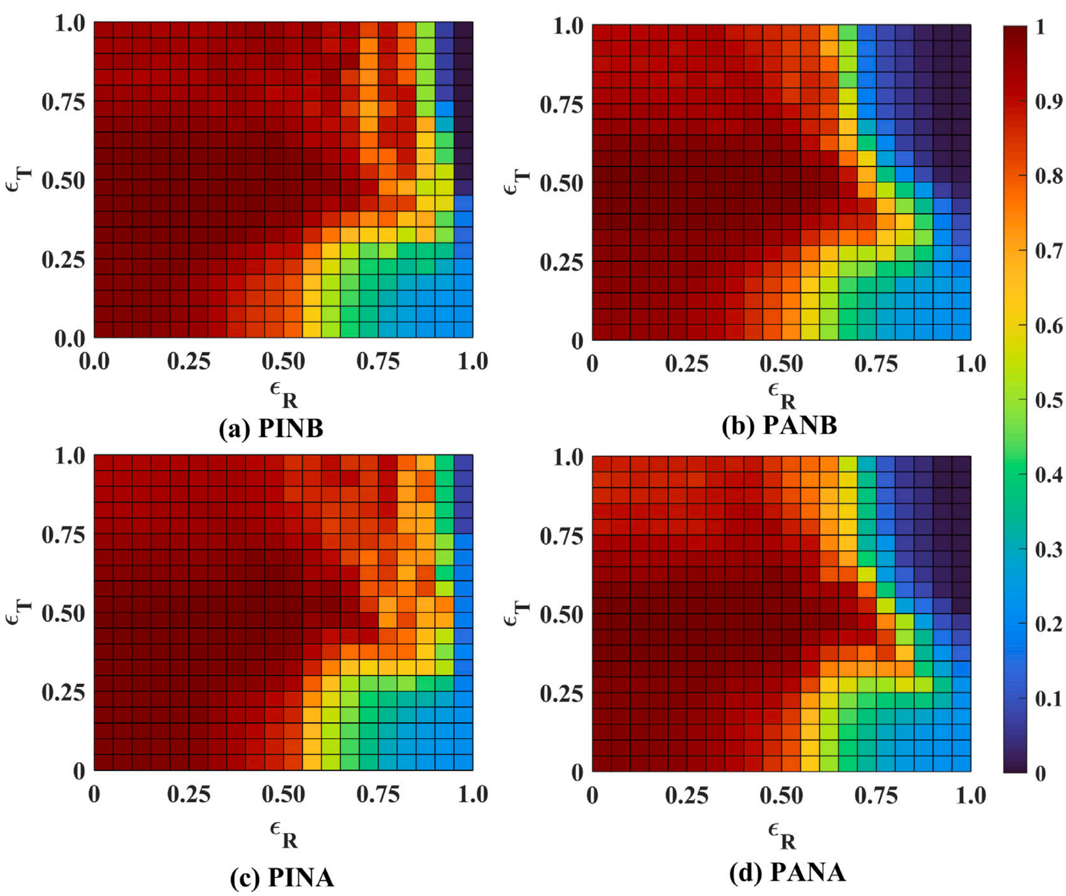

Fig. 4. Analysis of opinion evolution robustness in different signed networks with communities under random initial opinion distribution. The horizontal axis shows the repulsion threshold ranging from 0 to 1 with intervals of 0.05, while the vertical axis represents the assimilation threshold, also ranging from 0 to 1 with intervals of 0.05. The heat map shows the degree of bi-polarization, where deep red indicates maximum bi-polarization with two opposing groups, and blue represents complete consensus in the network. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

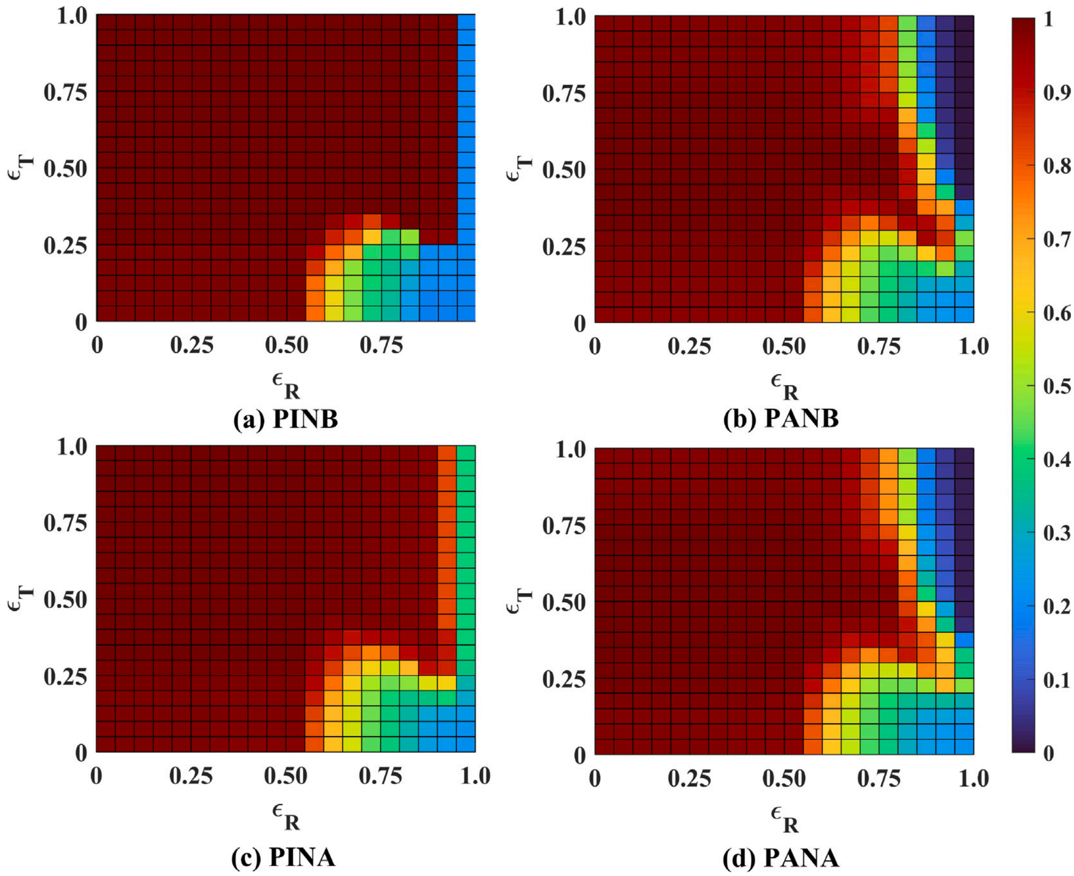

Fig. 5. Analysis of opinion evolution robustness in different signed networks with communities under biased initial opinion distribution. The horizontal axis shows the repulsion threshold ranging from 0 to 1 with intervals of 0.05, while the vertical axis represents the assimilation threshold, also ranging from 0 to 1 with intervals of 0.05. The heat map shows the degree of bi-polarization, where deep red indicates maximum bi-polarization with two opposing groups, and blue represents complete consensus in the network. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

indicates that in signed community networks, the combined effect of assimilative and repulsive social influences on opinion evolution is robust and exhibits a non-monotonic nature.

3.2. The impact of initial community opinion distribution

3.2.1. Influence of biased initial opinions between sub-communities

Analyzing the evolution of opinions within different signed social networks under conditions of biased initial opinion distributions across sub-communities reveals distinct outcomes. The design of the initial opinions between sub-communities is described in Section 2.3. Fig. 5 illustrates the distribution of bi-polarization degrees in signed networks with biased initial opinion distributions as opinions evolve to a steady state. For a detailed analysis of how different repulsion and assimilation thresholds influence opinion evolution in various signed network structures under fixed assimilation and repulsion thresholds, please refer to the Supplemental Information Fig. S1, S2 and S3.

From the heat map in Fig. 5, it is observed that when individuals in different sub-communities have initial opinions with biased tendencies towards opposition, the difficulty of achieving consensus in opinions increases compared to random initial opinion distributions within different signed networks with communities structures. Especially in PINB and PINA communities, the assimilation threshold shows almost non-monotonic effects on the degree of bi-polarization as opinions evolve to a steady state. Under fixed repulsion thresholds, increasing the assimilation threshold almost always promotes bi-polarization. However, at lower assimilation thresholds, specifically less than 0.3, increasing the repulsion threshold initially decreases and then increases the degree of bi-polarization at steady state. This phenomenon occurs because at higher repulsion thresholds, repulsive social influence allows time for opinions within sub-communities to converge, thereby facilitating bi-polarization. Therefore, when there are many antagonistic edges between communities and the threshold for repulsive social influence is high, lower trust levels weaken the degree of bi-polarization.

3.2.2. Comparative analysis of initial opinion distributions

Fig. 6 shows the differences between random and biased initial opinion distributions in sub-communities. Notably, in the PINB signed networks with communities, only under conditions of low assimilation thresholds and very high repulsion thresholds does the biased distribution slightly reduce the degree of bi-polarization compared to the random distribution. This is because, although the initial opinion distribution in sub-communities is biased, this somewhat limits the random interactions among individuals when the difference in opinions is at its maximum, thereby slightly reducing the degree of bi-polarization. Maintaining a diversity of group opinions between different communities can somewhat mitigate the formation of bi-polarization.

3.3. The role of node connection preferences in opinion dynamics

According to the community network structure construction method cited in this article, the parameter p_i controls the probability that an individual connects to other individuals within the same community, thus $1 - p_i$ is the probability of connecting to individuals in different communities.

In this section, the analysis focuses on how the tightness of internal connections, as controlled by p_i (which represents the probability of an

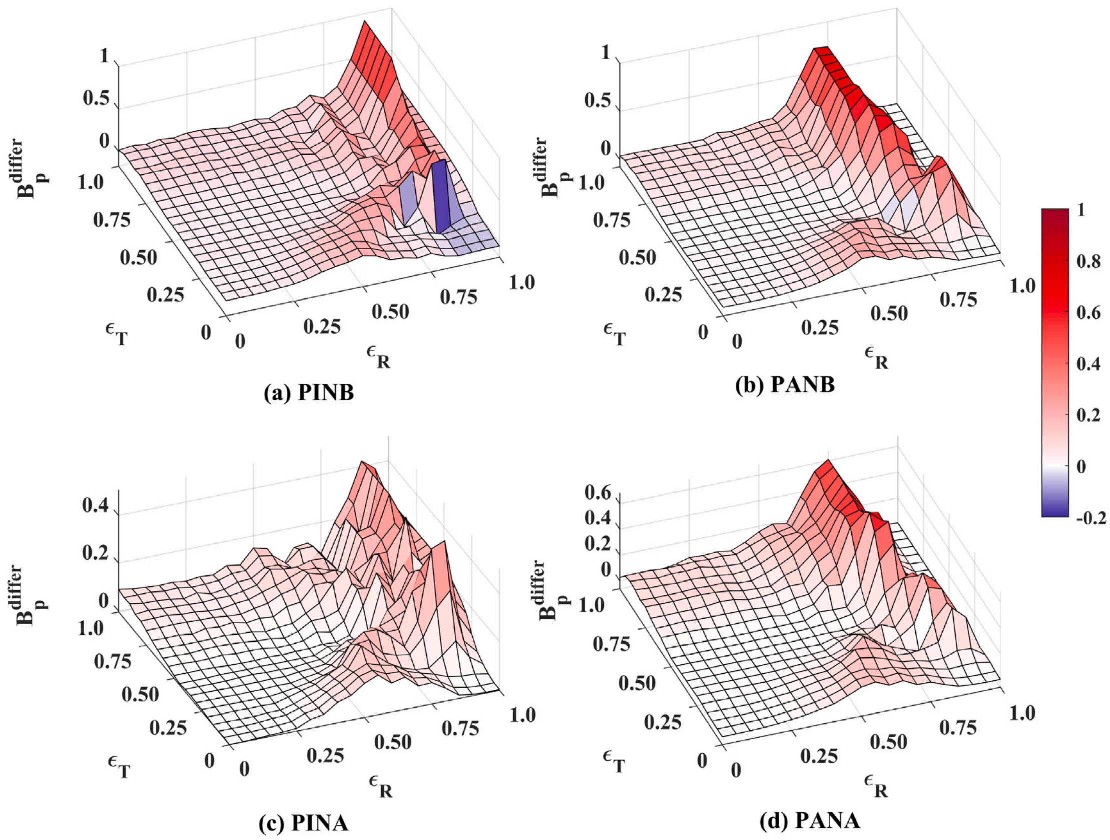

Fig. 6. Heat map showing the differences in the degree of bi-polarization (B_p) between random (Fig. 4) and biased (Fig. 5) initial opinion distributions. The x-axis represents the repulsion threshold, and the y-axis represents the assimilation threshold. The z-axis indicates the difference in the degree of bi-polarization (B_p) at steady state under the respective threshold conditions, with red indicating that the biased distribution promotes higher bi-polarization and blue indicating a reduction in bi-polarization compared to the random distribution. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

individual forming connections with others within the same community) affects the evolution of opinions across the entire network and the degree of bi-polarization at steady state. Figs. 7 and 8 respectively show the average results after conducting 20 independent simulation experiments. Additionally, the impact of the initial opinion distribution characteristics within sub-communities on the outcomes is compared and analyzed.

By fixing the repulsion threshold and adjusting the assimilation threshold, this study observed the influence of p_i, which controls the probability of individuals connecting within their own community, on the degree of bi-polarization at steady state in different signed networks with community structures. From Fig. 7, it can be observed that when initial opinions in the network are randomly distributed, increasing the probability of individuals connecting within their own community to 0.9 effectively reduces the degree of bi-polarization to some extent under both strong and weak repulsive social influences. However, it is noteworthy that increasing this probability does not always linearly decrease the degree of bi-polarization. For example, in the four sub-figures of Fig. 7(1), when the repulsion threshold is 0.75 and the assimilation threshold ranges from 0.1 to 0.4, the degree of bi-polarization at steady state increases as the probability of individuals connecting within their own community increases from 0.1 to 0.7. Similarly, when the assimilation threshold exceeds 0.5, the lack of connections between PINB and PINA communities increases the risk of bi-polarization. In PANB and PANA communities, effective inter-community communication and positive connections weaken the degree of bi-polarization. Under strong repulsive social influence, as shown in group Fig. 7(2), it is observed that only when the assimilation threshold is less than 0.3, increasing the probability of individuals connecting within their own community reduces the degree of bi-polarization. This indicates that under strong repulsive social influence, isolating repulsion between sub-communities and avoiding excessive convergence within communities to trigger inter-community repulsion mechanisms is one of the effective strategies to control the degree of bi-polarization.

Further analysis was conducted on the scenario where the initial opinions in sub-communities are biased, with the results presented in Fig. 8. It can be observed from Fig. 8 that in different types of signed networks with communities, when the repulsive social influence is weak and the initial opinion distribution in sub-communities shows opposing tendencies, the degree of bi-polarization at the steady state of opinion evolution increases with the increase in the probability of individuals connecting within their own community. Particularly, from Fig. 8(1b) and (1d), it is found that in PANB and PANA communities, when the probability of individuals connecting within their own community increases to 0.9 and the assimilative social influence exceeds 0.5, the degree of bi-polarization decreases. This phenomenon occurs because the assimilation threshold causes the opinions within the same community to converge quickly, while the positive connections between communities and the presence of strong assimilative social influence allow for effective communication between communities, facilitating convergence of opinions among some individuals. It is found that when considering the combined effects of assimilative and repulsive social influences on opinion evolution, one cannot simply assume that avoiding communication between different communities will effectively reduce the degree of bi-polarization. This is a complex process that should consider the combined effects of different factors based on specific situations. Therefore, this study provides a theoretical foundation from the micro-level to explain phenomena at the macro-level.

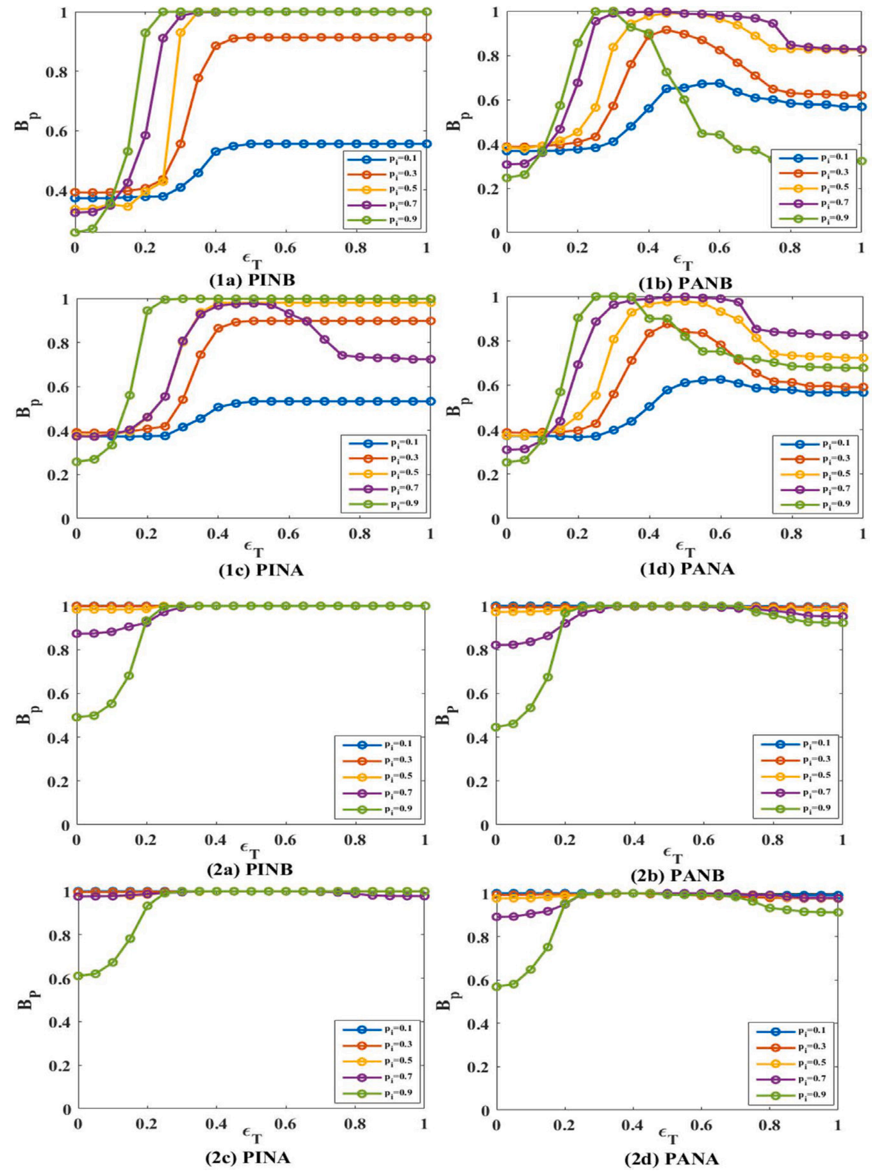

Fig. 7. The diagram illustrating the effect of node-connected edge preferences (p_i) with a random distribution of initial opinions on the opinion evolution. The node-connected edge preferences (p_i) are fixed at values of 0.1, 0.3, 0.5, 0.7, and 0.9, with different colored lines representing the corresponding degrees of bi-polarization (B_p) at steady state under varying assimilation threshold (ϵ_T) conditions. The analysis is conducted for two scenarios: (1a)–(1d) with a repulsion threshold set at 0.75, and (2a)–(2d) with a repulsion threshold set at 0.35. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3.4. Case studies on real-world signed social networks with communities

Based on these two real-world signed social network structures, the impact of different assimilation and repulsion threshold conditions on the degree of bi-polarization when network opinions evolve to a steady state was further explored. The results are shown in Fig. 9. These outcomes represent the mean of 20 independent simulation experiments under different initial opinions uniformly distributed randomly. The opinion evolution results for the Gahuku-Gama tribe are presented in Fig. 9(a), while those for the Slovenian parliamentary parties are shown in Fig. 9(b). The results are in general agreement with the evolutionary patterns in Fig. 4(b) and (c). A comparison of the two sub-figures reveals differences in opinion evolution across different signed social networks with communities.

In the Gahuku-Gama tribal network, when the repulsion threshold is less than 0.5, the degree of bi-polarization clearly increases with the assimilation threshold, and setting the assimilation threshold to its maximum value does not diminish the degree of bi-polarization. However, in the Slovenian parliamentary party network, when the repulsion threshold is below 0.5, the network's opinions evolve to a state of maximum bi-polarization at the steady state. Between repulsion thresholds of 0.5 and 0.75, the Gahuku-Gama tribe, compared to the Slovenian Parliamentary Party Network, shows a clear pattern where the degree of bi-polarization first increases and then decreases with rising assimilation thresholds, and within the range of 0.3 to 0.6 for the assimilation threshold, the degree of bi-polarization in the Gahuku-Gama tribe is higher than in the Slovenian Parliamentary Party Network. Therefore, when the assimilation threshold is low, it is advisable to increase the number of positive edges between communities; while when the assimilation threshold is high and the repulsion threshold is moderate, the presence of positive edges between communities poses a risk of promoting a higher degree of bi-polarization.

Fig. 10 shows the opinion evolution process for these two real signed networks when the assimilation threshold is 0.15 and the repulsion threshold is 0.4. The lines in the diagram represent the evolution of nodes with different initial opinions over time. In the PANB signed social networks with communities, the degree of bi-polarization is

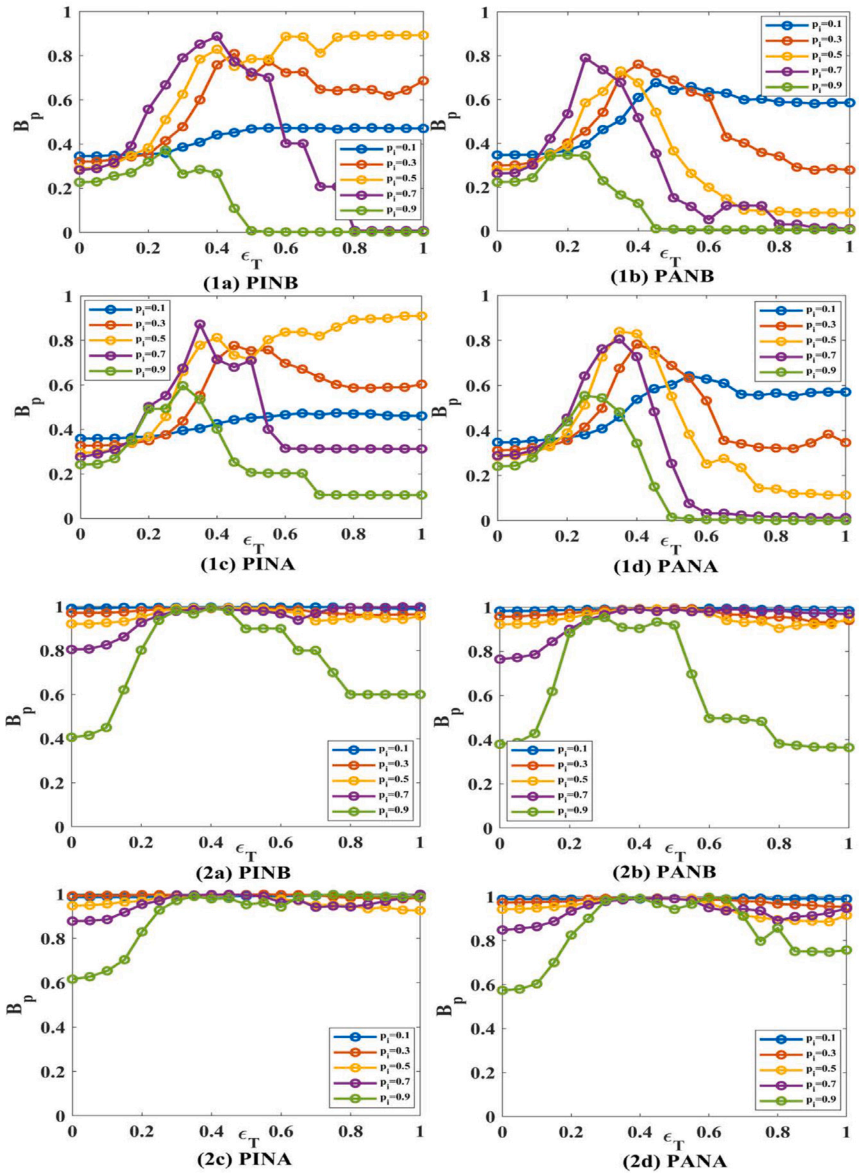

Fig. 8. The diagram illustrating the effect of node-connected edge preferences (p_i) with a biased distribution of initial opinions on the opinion evolution. The node-connected edge preferences (p_i) are fixed at values of 0.1, 0.3, 0.5, 0.7, and 0.9, with different colored lines representing the corresponding degrees of bi-polarization (B_p) at steady state under varying assimilation threshold (ϵ_T) conditions. The analysis is conducted for two scenarios: (1a)–(1d) with a repulsion threshold set at 0.75, and (2a)–(2d) with a repulsion threshold set at 0.35. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

less than that in the PINA signed social networks with communities. Positive connections effectively prevent the spread of repulsive social influences, resulting in the coexistence of multiple opinion clusters within the network.

4. Conclusion and discussion

The study delves into the complex interplay of assimilative and repulsive influences across different community structures within signed networks, highlighting the challenges in understanding and predicting opinion bi-polarization. Through simulation experiments based on signed networks with communities, this research sheds light on the dynamics of opinion evolution and proposes sociologically significant and practically applicable strategies.

The study reveals that positive connections within a community can foster consensus even with opposing initial opinions, contrasting with negative connections that increases the degree of bi-polarization. With weak repulsive social influence and random initial opinion distributions between communities, the assimilation threshold significantly influences bi-polarization, initially promoting it before weakening it. Biased initial opinions, coupled with strong repulsive social influence, can lead to opposing camps despite strong assimilative social influence, emphasizing the value of diverse initial opinions in reducing bi-polarization. Moreover, adjusting internal community interaction modes can effectively impact opinion bi-polarization across the network, highlighting the need to consider network structure and social influence in managing online platforms and addressing social divisions. Finally, the robustness of the results is verified through real-world signed network data.

These findings provide insights into predicting and managing opinion evolution in complex networks, offering strategies for reducing social divisions and fostering a more harmonious society. Future research could explore how different network topologies and social influences jointly affect opinion evolution and extend the study to more real-world networks to validate and enrich the findings.

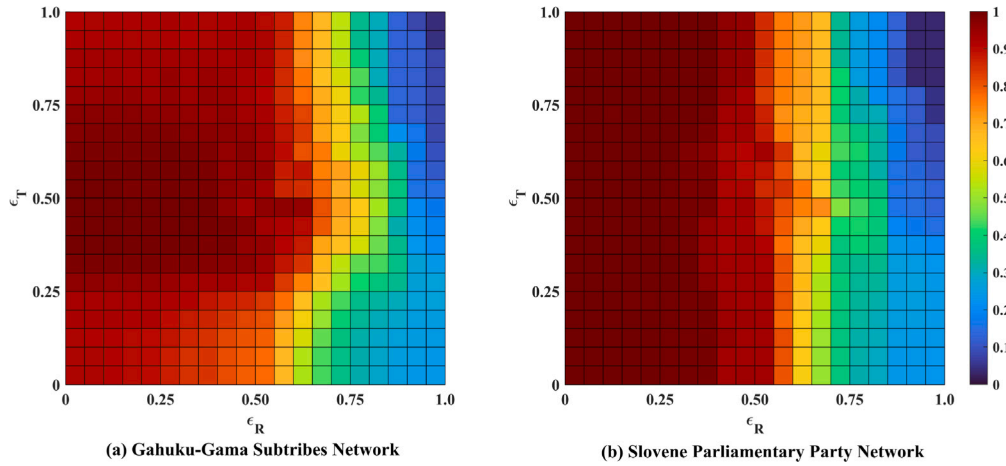

Fig. 9. The effect of assimilation and repulsion thresholds on opinion evolution in real signed networks. The left heat map shows the degree of bi-polarization (B_p) at steady state under varying thresholds in the Gahuku-Gama subtribes network, while the right heat map presents the same analysis for the Slovene Parliamentary Party network.

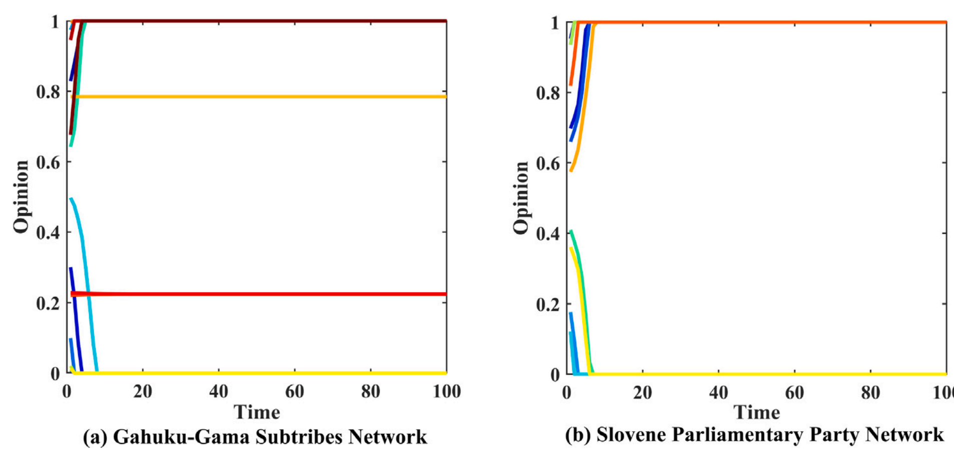

Fig. 10. The diagram of the opinion evolution process in the real signed networks with communities. The left panel represents the Gahuku-Gama Subtribes network, while the right panel depicts the Slovene Parliamentary Party network ($\epsilon_T = 0.15$, $\epsilon_R = 0.4$).

CRediT authorship contribution statement

Shuo Liu: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Shuhui Guo:** Writing – review & editing, Visualization. **Huijun Zheng:** Writing – review & editing. **Wenxuan Fu:** Writing – review & editing. **Haoxiang Xia:** Writing – review & editing, Writing – original draft, Supervision, Funding acquisition. **Xin Lu:** Writing – review & editing, Writing – original draft, Supervision, Investigation, Funding acquisition, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.chaos.2024.115735.

Data availability

No data was used for the research described in the article.

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